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THE STUDY STATE OF STRESS AND STRAINS BETWEEN DRILL PIECE AND CHIP

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Abstract: *It is determined the stress and strains state to the nose of the drill during the cutting process, using an analytical calculus model and MatchCAD software.*
First, it is considered the case of a linear contact between drill and work piece. Second, it is analyzed the stress state to the cylindrical contact between drill with rounded nose and work piece.

Key words: *drill nose, cutting, stress state, contact pressure, axial force.*

1. INTRODUCTION

Cutting of metals and other engineering materials has been and still is the major shaping process used in the production of engineering components.

In order to evaluate theoretically and experimentally the wear process of the cutting tools, it is necessary to know the state of stresses, the contact pressure, relative velocities between the tool and the work piece material.

Beginning from the cutting model in the drilling process, it is determined the variation of the stresses, the contact pressure and axial forces to the nose of the drill, using MathCAD software.

Every body accepts the hypothesis that the manufactured material present an active zone of the shearing with initial shear, at the nose of cutting tool, and the forming chip is moving along the rake face.

In a highly simplified form, the chip forming process is similar to the plowing mechanism. It is considered the slip-lines field associated with the rigid cutting wedge in contact with a rigid-plastic deformation of plane surface. The slip-lines field is accord with the Hencky's slip-lines theory [1 - 4].

It is considered the contact between the drill nose and plane surface of the work piece material (Fig.1). For materials manufacturing, the drill nose has the wedge form with β angle.

The initial contact length with plane surface of work piece material is *B*. For other materials, the nose of drill can have some different forms, for example wood working materials.

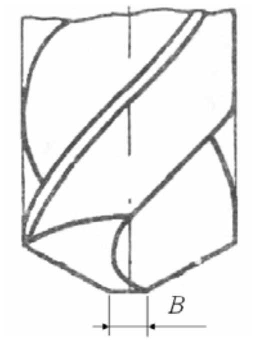

Fig.1. The contact model between drill and plane surface of work piece material.

2. THE STRESS STATE ON THE LINEAR CONTACT

It is analyze the stresses and elastically strains for the linear contact of a wedge form with β angle and *B* length (Fig.2).

It is considered a small β angle (sin β ≈ β). Thus, the elastically penetration of the tool in the work piece material *h*(*y*) can be writing under linear form [mm] [5]:

$$h(y) = \begin{cases} \delta - \varphi\, y & pentru \quad y > 0 \\ \delta + \varphi\, y & pentru \quad y < 0 \end{cases}, \tag{1}$$

and

$$\frac{dh}{dy} = -\varphi\, sgn(y), \tag{2}$$

where sgn (*y*) is:

$$sgn(y) = \begin{cases} -1 & pentru \quad y < 0 \\ 1 & pentru \quad y > 0 \end{cases}. \tag{3}$$

It is accepted that the elastically penetration of drill nose is making without friction, thus, the tangential stresses are 0, *g*(*y*) = 0.

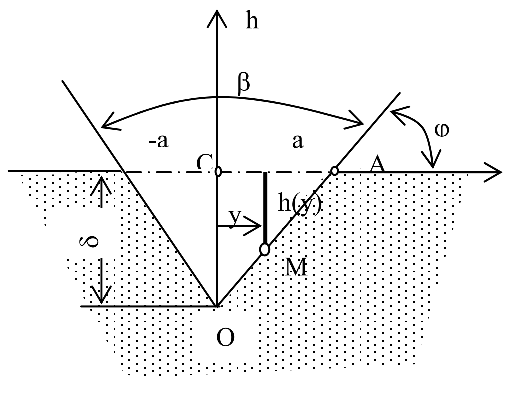

Fig.2. The model of tool (linear wedge).

The continuous contact condition has Hills form [5]:

$$\frac{1}{A_1}\frac{\partial h}{\partial y} = \frac{1}{\pi}\int \frac{p(\xi)d\xi}{y-\xi} - A_2 q(y), \qquad (4)$$

where:

$$A_1 = \frac{2(1-\nu_1^2)}{E_1} + \frac{2(1-\nu_2^2)}{E_2} = \frac{2}{E_2}\left[1-\nu_2^2 + \frac{1-\nu_1^2}{E_1/E_2}\right] = \frac{2}{E_2}E_{12}, \qquad (5)$$

$$A_2 = \frac{1}{2}\frac{[(1+\nu_1)(1-2\nu_1)/E_1 - (1+\nu_2)(1-2\nu_2)]/E_2}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}, \qquad (6)$$

A_1 - the elasticity parameter of drill material (Poisson coefficient υ_1, elasticity longitudinal modulus E_1); A_2 – the elasticity parameter of workpiece material (υ_2, E_2); $E_{1,2}$ - the relative elasticity between workpiece material and drill material [N/mm^2]:

$$E_{12} = 1-\nu_2^2 + \frac{(1-\nu_1^2)E_2}{E_1}. \qquad (7)$$

Because the drill material is harder than the piece material, $E_2/E_1 = 0$, $E_{12} = 1-\nu_2^2$.

The contact pressure in a point M, caracterized by nondimensional coordinate $s = y/a$ ($2a$ - the contact weidth of drill nose), become[5]:

$$p(s) = -\frac{\varphi}{\pi A_1}\left(\int_0^1\left(\frac{1}{r-s}+\frac{1}{r+s}\right)\frac{dr}{\sqrt{1-r^2}}\right)\sqrt{1-s^2}$$

$$= -\frac{2\varphi}{\pi A_1}\sqrt{1-s^2}\int_0^1\frac{r\,dr}{(r^2-s^2)\sqrt{1-r^2}} =$$

$$= -\frac{2\varphi}{\pi A_1}\arg\cos h(a/|y|) \qquad (8)$$

The cutting force (F_x) (Fig.3) it is determined from the condition of mechanical equilibrium:

$$F_x = B\int_{-a}^{a} p(y)dy, \qquad (9)$$

$$a = \frac{F_x A_1}{2B\varphi} = \frac{F_x \cdot A_1}{B(\pi-\beta)}. \qquad (10)$$

Thus, the nondimensional contact pressure on the wedge of cutting tool becomes [6]:

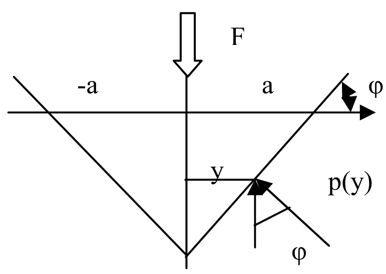

Fig.3. The pressure distribution on the drill nose.

$$p_a(s,\beta,E_{12}) = -\frac{\pi-\beta}{\pi E_{12}}\arg\cosh\left(\frac{1}{|s|}\right). \qquad (11)$$

In Fig.4, it is presented the pressure variation for different β angles, function of relative position of contact point, *s*.

For the cutting process, the maximum pressure to the drill nose is limited to critical pressure.

The critical pressure (p_c) is defines like a pressure which determined a plastically deformation from a shearing plane of cutting.

The determination of critical pressure is based from Hencky's slip-lines theory [N/mm^2] [6]:

$$p(s) = p_c = -\frac{2\varphi}{\pi A_1}\arg\cos h(1/|s|) \qquad (12)$$

and

$$p_{ca} = \frac{p_c}{E_2} = \frac{(\beta-\pi)}{\pi E_{12}}\arg\cos h\left(\frac{1}{|s|}\right). \qquad (13)$$

The numerical solution of equation (13), with unknown *s*, is presents in Fig. 5 (for different β angles, relative elasticity E_{12} and resistance conditions p_c/E_2).

With this relation, can be determined the axial force necessary for cutting generation, F_x.

The nondimensional pressure distribution to the drill nose, without friction and moving, is:

$$p_a(s) = \begin{cases} p_{ca} & pentru \quad s < |s_0| \\ -\frac{\beta-\pi}{\pi E_{12}}\arg\cos h\left(\frac{1}{|s|}\right) & pentru \quad |s_0| \le s \le 1 \end{cases} \qquad 14)$$

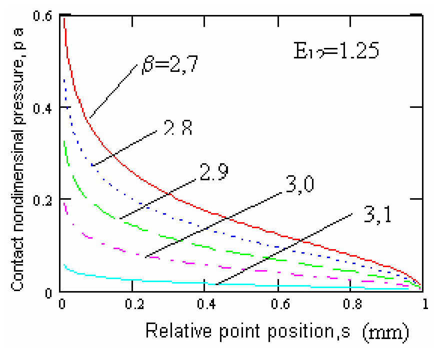

Fig.4. The contact pressure variation on the drill wedge.

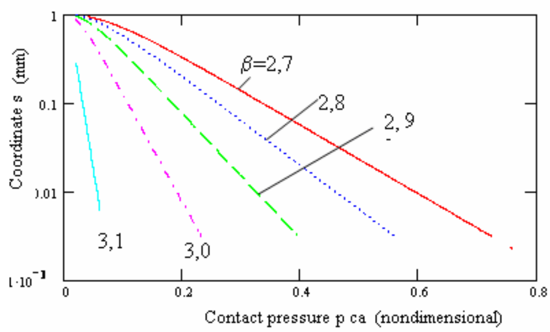

Fig.5. Critically coordinates of maximum pressure point.

and

$$F_x = 2B|s_0|a\ p_{ca}E_2 + \\ + 2B\ a\ E_2 \frac{-\beta+\pi}{\pi E_{12}} \int_{|s_0|}^{1} arg\cosh\left(\frac{1}{|s|}\right) ds, \quad (15)$$

$$F_{a_x} = \frac{F_x}{2B\ a\ E_2} = |s_0| p_{ca} + \frac{-\beta+\pi}{\pi E_{12}} \int_{|s_0|}^{1} arg\cosh\left(\frac{1}{|s|}\right) ds. \quad (16)$$

where: F_{ax} - nondimensional force, necessary to generate the cutting process

In Fig.6 it is presented the variation of axial nondimensional force (F_{ax}) function of different β angles of tool nose, and some values of nondimensional critical pressure (p_{ca}).

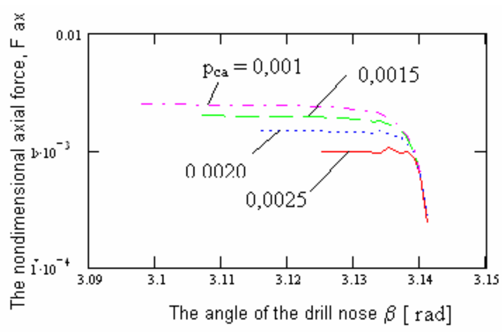

Fig.6. Variation of nondimensional axial force (F_{ax}).

3. THE STRESSES STATE ON THE CYLINDRICAL CONTACT

It is considered the cylindrical initial contact between drill nose and piece material (Fig. 7).

The initial elastically deformation of drill nose $h(y)$ it is associate with a second degree curve [mm]:

$$h(y) = \delta - ky^2/2, \quad (17)$$

where: δ – maximum penetration [mm]; $k = 1/R_1 + 1/R_2$ - total curving; R_1 – curving radius of drill nose [mm]; R_2 – curving radius of work piece material [mm].

When to the contact area finding the normal pressures $p(y)$ and tangential pressures $q(y)$, the strain $h(y)$ become [5, 7, 8]:

$$\frac{1}{A_1}\cdot\frac{\partial h}{\partial y} = \frac{1}{\pi}\int\frac{p(\xi)d\xi}{y-\xi} - A_2 q(y), \quad (18)$$

where A_1, A_2 - elasticity parameters

If the size of contact surface is determined only by normal load, the limits of equation (14) are known: $-a$, a.

If the tangential stresses are generated by friction, $q(y) = f{\cdot}p(y)$, f - friction coefficient, the equatioin (18) is by second degree, Cauchy type.

The solution become [N/mm²] [5]:

$$p(s) = -\frac{ka}{A_1}(1-s)^m(1+s)^{1-m}\sin m\pi \\ = -p_0(1-s)^m(1+s)^{1-m} \quad (19)$$

function of β angle.

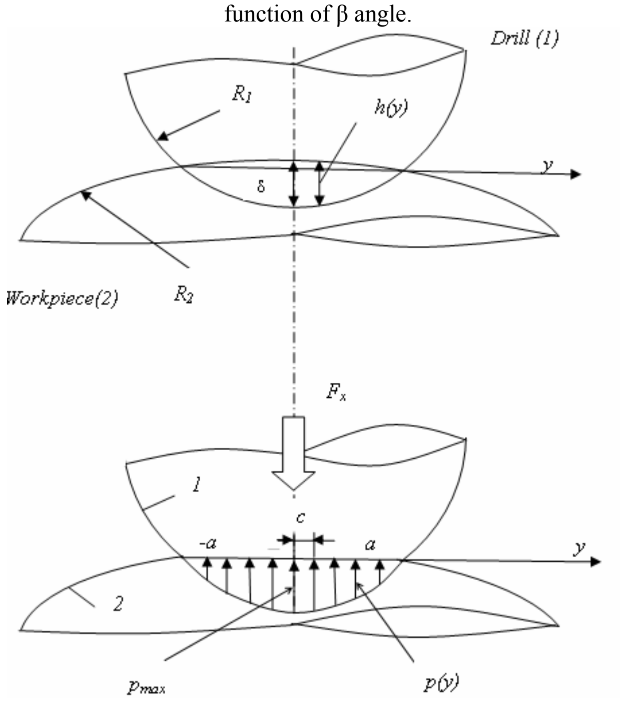

Fig.7. The contact between drill nose and curve surface of work piece material.

where: $m = arctg\left(\frac{1}{A_2 f}\right)$, $0 < m < 1$

$$s = a/y; \quad p_0 = \frac{ka}{A_1}\sin m\pi.$$

The axial force will be [6]:

$$F_x = p_0\int_{-a}^{a}\left(1-\frac{y}{a}\right)^m\left(1+\frac{y}{a}\right)^m dy = \frac{2p_0\pi am(1-m)}{\sin m\pi}. \quad (20)$$

where: F_x – the force on the drill axis necessary to cutting generation [N] ($2a$-the chip depth [mm]).

Between maximum contact pressure (p_0), chip depth ($2a$), and axial force (F_x) the next relations are establish:

$$p_0 = \frac{F_x\sin m\pi}{2\pi am(1-m)}, \quad (21)$$

$$a^2 = \frac{F_x A_1}{2\pi m(1-m)k}. \quad (22)$$

The nondimensional pressure distribution is:

$$p_a(s,k,a,f,E_{12}) = \frac{p(s)}{E_2} = \frac{1}{2}\frac{ka}{E_{12}}\sin m\pi(1-s)^m(1+s)^{1-m}. \quad (23) \\ = p_{a0}(1-s)^m(1+s)^{1-m}$$

where: $p_{a0} = \frac{1}{2}\frac{ka}{E_{12}}\sin m\pi$ - nondimensional maximum pressure on drill nose.

In Fig.8 it is presented the pressure variation (p_a) on the drill nose, function of some paramteres (f, s).

From analyze of $p_a(s)$ with m parameter, it is obtained the maximum function:

$$p_{a\,max} = p_{a0}\left\{(2m)^m + [2(1-m)]^{1-m}\right\} \qquad (24)$$

for:

$$s = 1 - 2m\,. \qquad (25)$$

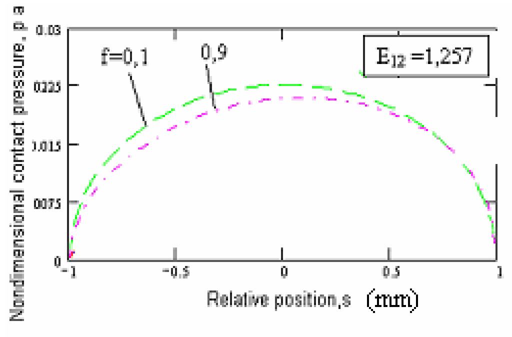

Fig.8. The contact pressure variation on the drill wedge.

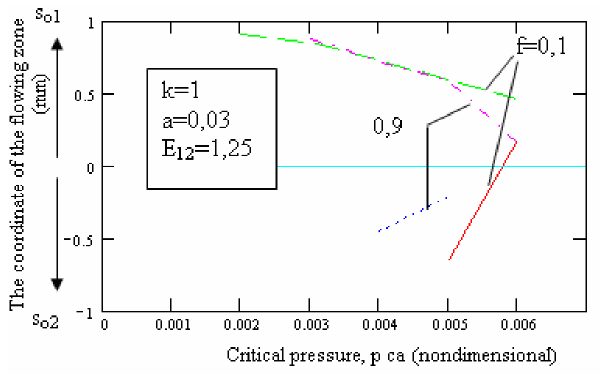

Fig.9. The coordinates of the flowing zone at the nose of the cutting tool.

Using the equation (23) can determined he length of the chip flowing, at the nose of the cutting tool:

$$\{s_{01}, s_{02}\} = solve\ eq(p_a - p_{ca} = 0)\,, \qquad (26)$$

with condition: $p_a(s,k,a,E_{12}) = \dfrac{p_c}{E_2} = p_{ca}\,.$

The numerical solutin of equation (26) is presents in Fig. 9 (for different cutting parameters):

If the critical pressure p_c it is realized, then, the pressure distribution on the tool nose is:

$$p_a = \begin{cases} p_{a0}(1-s)^m(1+s)^{1-m} & pentru \quad s \in (-1, s_{01}) \cup (s_{02}, 1) \\ p_{ca} \quad pentru & s \in (s_{01}, s_{02}) \end{cases}, \qquad (27)$$

where s_{01} and s_{02} are the solutions of equation (26).

Nondimensional pressure to the nose of cutting tool increases linear with the angle of tool β and decreases with relative elasticity of the work piece material.

If the nondimensional critical pressure p_{ca} is better than $p_{a\,max}$, the cutting process can't be realized.

4. CONCLUSIONS

Regarding analyze of the drilling process, the main contributions of the authors are the following:

1. Using the theory about initial elastic contact of the drill nose, it is explained the chip apparition and the stresses distribution, for different nose geometry: elastic angular wedge, cylindrical rounded nose;
2. Beginning from analyze of stresses in case of elastically tool deformations, it is determined the nondimensional pressure variation and the nondimensional axial force, necessary for generate the cutting process; each of them for different case of tool nose geometry;
3. It is determined the stress state and deformations on the conical edge of drill, using the Hencky-Mises slip lines theory;
4. It is determined the contact length of the cutting tool, function of drill geometry and working conditions.

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